Radiative neutrino transition $\nu \to \nu\gamma$ in strongly magnetized plasma

M.V. Chistyakov*, N.V. Mikheev†

Division of Theoretical Physics, Department of Physics,

Yaroslavl State University, Sovietskaya 14,

150000 Yaroslavl, Russian Federation

Abstract

The influence of strongly magnetized electron-positron plasma on the radiative neutrino transition $\nu \to \nu\gamma$ is investigated. The probability and mean losses of the neutrino energy and momentum are calculated taking account of the photon dispersion and large radiative corrections near the resonance. It is shown that the combined effect of plasma and strong magnetic field decreases the probability and mean values of the neutrino energy and momentum loss in comparison with these values obtained in pure magnetic field.

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*E-mail address: mch@univ.uniyar.ac.ru
†E-mail address: mikheev@yars.free.net

It is well known that neutrino processes play an important role in astrophysics [1]. In many cases these processes occur in the presence of plasma and/or magnetic field. Of particular conceptual interest are those processes which are forbidden or strongly suppressed in vacuum. One such reaction is the radiative massless neutrino transition $\nu \to \nu\gamma$. Previously this process was studied in plasma and magnetic field separately. In plasma the process $\nu \to \nu\gamma$ was firstly investigated in [2] and more recently in [3, 4]. In pure magnetic field the radiative neutrino transition $\nu \to \nu\gamma$ was studied in the papers [5, 6, 7, 8]. In the framework of four-fermion theory the amplitude and the probability of the process were calculated in refs. [5] and [6] in the crossed field and strong magnetic field respectively. In the Standard Model the amplitude of the neutrino transition $\nu \to \nu\gamma$ was found in [7, 8] for the arbitrary magnetic field strength. In the paper [7] the case of the moderate neutrino energies, $E_\nu < 2m_e$ was studied in the kinematical region where the final photon dispersion law was closed to the vacuum one, $q^2 = 0$. The limit of the large neutrino energies and strong magnetic field was investigated in [8]. There is that case which could be realized at the Kelvin-Helmholz stage of supernova remnant cooling, when the energies of the neutrino are $E_\nu \simeq 10-20$ MeV and the magnetic field strength could be as high as $10^{16} - 10^{17}$ G [9]. It was shown in [8] that the main contribution into the probability of the neutrino transition $\nu \to \nu\gamma$ was determined from the vicinity of the lowest cyclotron resonance, when the amplitude of the process and photon polarization operator contained simultaneously the square-root singularity.

The purpose of our work is to study the influence of the electron-positron plasma on the process of the radiative massless neutrino transition $\nu \to \nu\gamma$ in a strong magnetic field. This process is considered in the framework of the Standard Model using the effective local Lagrangian of the neutrino-electron interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\alpha(g_V - g_A\gamma_5)e]\, j_\alpha\,, \tag{1}$$

where $g_V = \pm 1/2 + 2\sin^2\theta_W$, $g_A = \pm 1/2$. Here the upper signs correspond to the electron neutrino ($\nu = \nu_e$) when both Z and W boson exchange takes part in a process. The lower signs correspond to μ and τ neutrinos ($\nu = \nu_\mu, \nu_\tau$), when the Z boson exchange is only presented in the Lagrangian (1), $j_\alpha = \bar{\nu}\gamma_\alpha(1-\gamma_5)\nu$ is the left neutrino current. We investigate the limit of ultrarelativistic strongly magnetized plasma, when the magnetic field strength is the largest physical parameter

$$eB > E_\nu^2,\, \mu^2,\, T^2 \gg m_e^2. \tag{2}$$

Here μ is the electron chemical potential, T is the temperature of plasma. Under

these conditions electrons and positrons in plasma occupy dominantly the lowest Landau level.

Notice that the amplitude and the probability of the process $\nu \to \nu\gamma$ depend essentially on the polarization of the final photon. In a general case there exist three eigenmodes of the photon polarization operator. The corresponding eigenvectors can be written in the following form:

$$\varepsilon_\mu^{(1)} = \frac{(q\varphi)_\mu}{\sqrt{q_\perp^2}}; \qquad \varepsilon_\mu^{(2)} = \frac{(q\tilde{\varphi})_\mu}{\sqrt{q_\parallel^2}}; \qquad \varepsilon_\mu^{(3)} = \frac{q^2 (q\varphi\varphi)_\mu - q_\mu (q\varphi\varphi q)}{\sqrt{q^2 q_\parallel^2 \, q_\perp^2}}, \tag{3}$$

where $\varphi_{\alpha\rho} = F_{\alpha\rho}/B$ is the dimensionless tensor of the external magnetic field, $\tilde{\varphi}_{\alpha\rho} = \frac{1}{2}\varepsilon_{\alpha\rho\mu\nu}\varphi_{\mu\nu}$ is the dual tensor, $q_\parallel^2 = (q\tilde{\varphi}\tilde{\varphi}q) = q_\alpha \tilde{\varphi}_{\alpha\rho}\tilde{\varphi}_{\rho\mu} q_\mu$, $q_\perp^2 = (q\varphi\varphi q)$. Only two of these modes, $\varepsilon_\mu^{(1)}$ and $\varepsilon_\mu^{(2)}$ are the physical one in the pure magnetic field. [1] As the analysis shows the presence of the strongly magnetized plasma doesn't modify the eigenvectors (3) but modifies the eigenvalue corresponding to the vector $\varepsilon_\mu^{(2)}$ only. This is due to the fact that the interaction of the two other eigenmodes with the electrons and positrons which occupy the lowest Landau level is strongly suppressed under the condition (2). Hence, only the photon with eigenvector $\varepsilon_\mu^{(2)}$ can be created in the process under consideration, as it takes place in the pure magnetic field [8].

The process of the radiative neutrino transition is depicted by the Feynman diagram, see Fig. 1, where the double line corresponds to the propagator of electron in the presence of a magnetic field and plasma. Several methods are known in literature describing the process in the background plasma. In the present paper we use the Real Time Formalism (RTF). The general expression of the real-time propagator in an external field can be found in the paper [11]. In the limit of a strong magnetic field the propagator can be presented in the form:

$$S(x,y) = e^{i\Phi(x,y)} \int \frac{d^4 p}{(2\pi)^4} \, S(p) \, e^{-ip(x-y)} \tag{4}$$

where

$$S(p) \simeq 2(\gamma p_\parallel + m)\Pi_- e^{-\frac{p_\perp^2}{eB}} \left(\frac{1}{p_\parallel^2 - m^2 + i\varepsilon} - 2i\pi f_F(p_0)\, \delta(p_\parallel^2 - m^2) \right), \tag{5}$$

$$f_F(p_0) = f_-(p_0)\theta(p_0) + f_+(-p_0)\theta(-p_0), \qquad \Pi_- = \frac{1}{2}(1 - i\gamma_1\gamma_2).$$

[1]The modes $\varepsilon_\mu^{(1)}$ and $\varepsilon_\mu^{(2)}$ correspond to Adler's so-called parallel ($\|$) and perpendicular ($\perp$) modes [10].

In (4) $f_{\mp}(p_0)$ are the distribution functions of electron and positron in plasma

$$f_{\mp}(p_0) = \frac{1}{e^{\frac{p_0 \mp \mu}{T}} + 1}.$$

Notice that in the case of two-point function the translationally and gauge noninvariant phase factors $\Phi(x, y)$ are cancelled:

$$\Phi(x, y) + \Phi(y, x) = 0.$$

Using the propagator (4) the amplitude of the process can be presented in the form:

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_{pl}, \tag{6}$$

where $\mathcal{M}_B$ is the amplitude of the process $\nu \to \nu\gamma$ corresponding to the pure magnetic field contribution ($T = \mu = 0$). Following [8] it can be expressed in the form [2]

$$\begin{aligned} \mathcal{M}_B &= \frac{eG_F}{2\pi^2\sqrt{2}} \frac{eB}{\sqrt{q^2}} \{g_V(j\tilde{\varphi}q) + g_A(jq)\} H\left(\frac{4m_e^2}{q^2}\right), \\ H(z) &= \frac{z}{\sqrt{z-1}} \arctan \frac{1}{\sqrt{z-1}} - 1, \; z > 1, \\ H(z) &= -\frac{1}{2}\left(\frac{z}{\sqrt{1-z}} \ln \frac{1+\sqrt{1-z}}{1-\sqrt{1-z}} + 2 + i\pi \frac{z}{\sqrt{1-z}}\right), \; z < 1. \end{aligned} \tag{7}$$

It should be noted that $\mathcal{M}_B$ is the amplitude with the definite photon polarization corresponding to the mode $\varepsilon_\mu^{(2)}$ from the equation (3).

The second term in (6), $\mathcal{M}_{pl}$, is induced by the coherent neutrino scattering on plasma electrons and positrons with photon radiation. For $\mathcal{M}_{pl}$ we find

$$\mathcal{M}_{pl} = -\frac{eG_F}{\pi^2\sqrt{2}}(eB)m_e^2\sqrt{q^2}\,\{g_V(j\tilde{\varphi}q) + g_A(jq)\} \int \frac{dp_z}{E} \frac{f_-(E) + f_+(E)}{4(pq)^2 - (q^2)^2}. \tag{8}$$

As was mentioned above the amplitude $\mathcal{M}_B$ contains the square- root singularity which is connected with the cyclotron resonance on the lowest Landau level. In the vicinity of the resonance point $q^2 = 4m_e^2$ it becomes:

$$\mathcal{M}_B \simeq \frac{eG_F}{4\pi\sqrt{2}} \frac{eB}{\sqrt{4m_e^2 - q^2}} \{g_V(j\tilde{\varphi}q) + g_A(jq)\}. \tag{9}$$

[2]Hereafter all expressions contain the scalar production of 4-vectors in Minkowski subspace (0, 3) only (field **B** is directed along the third axis). The metric of the subspace is defined by $g_{\parallel} = \tilde{\varphi}\tilde{\varphi} = (+, 0, 0, -)$. Therefore, for arbitrary 4-vectors a_μ, b_μ one has $(ab) = (a\tilde{\varphi}\tilde{\varphi}b) = a_0b_0 - a_3b_3$.

It is particularly remarkable that the amplitude $\mathcal{M}_{pl}$ contains the singularity of the same type. In the limit $q^2 \to 4m_e^2$ the total amplitude (6) can be presented in the following form:

$$\mathcal{M} \simeq \mathcal{M}_B \, \mathcal{F}(q_0), \tag{10}$$

where

$$\mathcal{F}(q_0) = \frac{\sinh x}{\cosh x + \cosh \eta}, \qquad x = \frac{|q_0|}{2T}, \;\; \eta = \frac{\mu}{T}.$$

It should be stressed that not only the amplitude $\mathcal{M}$ has the singular behaviour but the photon polarization $\mathcal{P}^{(2)}$ as well. It can be obtained from (10) by the following replacements

$$\mathcal{P}^{(2)} = -\mathcal{M}(\frac{G_F}{\sqrt{2}} g_V \to e, g_A \to 0, j_\alpha \to \varepsilon_\alpha^{(2)}).$$

For $\mathcal{P}^{(2)}$ one has:

$$\mathcal{P}^{(2)} \simeq -\frac{2\alpha e B m_e}{\sqrt{4m_e^2 - q^2}} \, \mathcal{F}(q_0). \tag{11}$$

A large value of $\mathcal{P}^{(2)}$ near the resonance requires taking account of large radiative corrections which reduce to a renormalization of the photon wave function:

$$\varepsilon_\alpha^{(2)} \to \varepsilon_\alpha^{(2)} \sqrt{Z}, \quad Z^{-1} = 1 - \frac{\partial \mathcal{P}^{(2)}}{\partial q^2}. \tag{12}$$

Using the formula (12) for the amplitude we find

$$\mathcal{M} \to \sqrt{Z}\mathcal{M} \simeq \frac{eG_F}{4\pi} \frac{eB}{\sqrt{q_\perp^2}} \{g_V (j\tilde{\varphi}q) + g_A (jq)\} \mathcal{F}(q_0). \tag{13}$$

Thus, the photon wave-function renormalization corrects the singular behaviour of the amplitude.

The probability of the process $\nu \to \nu\gamma$ can be obtained by integration of the amplitude over the phase space with taking account of the photon dispersion $q^2 - q_\perp^2 = \mathcal{P}^{(2)}$.

$$\begin{aligned} E_\nu W &= \frac{1}{32\pi^2} \int |\mathcal{M}\sqrt{Z}|^2 \, \delta(E_\nu - E_{\nu'} - q_0(\mathbf{k} - \mathbf{k}')) \times \\ &\times \frac{1}{1 - e^{-q_0/T}} \cdot \frac{d^3k'}{E_{\nu'} q_0} \end{aligned} \tag{14}$$

We assume that the neutrino distribution is closed to the Boltzman one, so one can neglect the deviation of the neutrino statistical factor from the unity. The probability (14) is rather complicated in the general case and it will be published in the extended paper. Here we present the results of our calculation in two limiting cases of the cold, $\mu \gg T$, and hot, $T \gg \mu$, plasma. Notice that in the vicinity of the cyclotron resonance, which gives the main contribution to the probability, the photon dispersion has a rather simple form $q_0 \simeq \sqrt{q_3^2 + 4m_e^2}$. In the limit of the low temperature, $\mu \gg T$, for the probability we obtain:

$$
\begin{aligned}
W_{LT} &\simeq \frac{\alpha (G_F e B)^2}{16\pi^2} E_\nu \Big\{ (g_V - g_A)^2 [(1-\lambda^2) - \frac{4\mu}{E_\nu}(1+\lambda)] \theta(1 - \lambda - \frac{4\mu}{E_\nu}) + \\
&+ (g_V + g_A)^2 \, [(1-\lambda^2) - \frac{4\mu}{E_\nu}(1-\lambda)] \theta(1 + \lambda - \frac{4\mu}{E_\nu}). \Big\}.
\end{aligned}
\tag{15}
$$

Here λ is the cosine of the angle between the initial neutrino momentum $\mathbf{k}$ and the magnetic field direction. In the opposite limit of high temperature, $T \gg \mu$, the result for the probability of the process $\nu \to \nu\gamma$ is:

$$
\begin{aligned}
W_{HT} &\simeq \frac{\alpha (G_F e B)^2}{4\pi^2} T \Big\{ (g_V - g_A)^2 (1+\lambda) \, F_1 \left(\frac{E_\nu (1-\lambda)}{8T} \right) + \\
&+ (g_V + g_A)^2 (1-\lambda) \, F_1 \left(\frac{E_\nu (1+\lambda)}{8T} \right) \Big\},
\end{aligned}
\tag{16}
$$

$$
F_1(x) = x + \ln(\cosh x) - \frac{1}{4} \tanh^2 x - \tanh x
$$

In the limit of the rarefied plasma both expressions (15) and (16) reproduce the known formula for the radiative neutrino transition probability in the pure strong magnetic field [8]:

$$
W_B \simeq \frac{\alpha (G_F e B)^2}{8\pi^2} (g_V^2 + g_A^2) E_\nu (1 - \lambda^2). \tag{17}
$$

In the case of the electron neutrino ν_e the dependence of the probabilities (15), (16), (17) on the initial neutrino energy are presented in Fig. 2, Fig. 3.

Keeping in mind a possible application of our results in astrophysics we calculate the mean values of the neutrino energy and momentum losses. These values could be defined by the four-vector:

$$
Q_\mu = E_\nu \int dW \cdot q_\mu = -E_\nu (\frac{dE_\nu}{dt}, \frac{d\mathbf{P}_\nu}{dt}),
$$

where zero component Q_0 of this vector is connected with the mean neutrino energy loss per unit time, the spatial components $\mathbf{Q}$ are connected with the momentum loss per unit time.

For the zero and third components of the Q_μ we obtain the following expression in the limit of cold plasma, $T \ll \mu$:

$$
\begin{aligned}
Q_{0,3} &\simeq \frac{\alpha (G_F e B)^2}{64\pi^2} E_\nu^3 (1-\lambda^2) \Big\{ (g_V + g_A)^2 \, [(1+\lambda) - \frac{16\mu^2}{E_\nu^2 (1+\lambda)}] \, \theta(1+\lambda - \frac{4\mu}{E_\nu}) \\
&\pm (g_V - g_A)^2 \, [(1-\lambda) - \frac{16\mu^2}{E_\nu^2 (1-\lambda)}] \, \theta(1-\lambda - \frac{4\mu}{E_\nu}) \Big\}. \qquad (18)
\end{aligned}
$$

In the opposite case, when $\mu \ll T$ we find

$$
\begin{aligned}
Q_{0,3} &\simeq \frac{\alpha (G_F e B)^2}{2\pi^2} E_\nu T^2 \Big\{ (g_V + g_A)^2 (1-\lambda) \, F_2 \left(\frac{E_\nu (1+\lambda)}{4T} \right) \\
&\pm (g_V + g_A)^2 (1+\lambda) \, F_2 \left(\frac{E_\nu (1-\lambda)}{4T} \right) \Big\}, \qquad (19)
\end{aligned}
$$

$$
F_2(x) = \frac{1}{2} \tanh \frac{x}{2} - \frac{x e^x (1 + 2e^x)}{(1+e^x)^2} + (2+x) \ln(1+e^x) + Li_2(-e^x) - \ln 4 + \frac{\pi^2}{12},
$$

where $Li_2(x)$ is the polylogarythm function. Notice that in the limit $T \to 0$, $\mu \to 0$ both expressions (18) and (19) reproduce the formula for the four-vector of losses in the pure strong magnetic field [8]:

$$
Q_{0,3} = \frac{\alpha (G_F e B)^2}{64\pi^2} E_\nu^3 (1-\lambda^2) \Big\{ (g_V + g_A)^2 (1+\lambda) \pm (g_V - g_A)^2 (1-\lambda) \Big\}. \qquad (20)
$$

We note, that electron-positron plasma and photon gas make an opposite influence on the process under consideration. On one hand, the electron-positron background decreases the amplitude of the process ($\mathcal{F}(q_0) < 1$). On the other hand, the probability and the mean value of the neutrino energy and momentum loss increases due to the effect of the stimulated photon emission. The analysis shows that the combined effect of electron-positron plasma and photon gas leads to the decreasing of the probability in comparison to the result in the strong magnetic field (17) as one can see from the Fig. 2, Fig. 3. The similar supressing plasma influence on four-vector of neutrino energy and momentum losses takes place.

In conclusion, we have calculated the process of the radiative neutrino transition in the presence of plasma and strong magnetic field. It was shown that the combined

effect of plasma and strong magnetic field decreases the probability and mean values of neutrino energy and momentum loss in comparison with these values obtained in pure magnetic field. Therefore the complex medium plasma + strong magnetic field is more transparent to neutrino with regard to the process $\nu \to \nu\gamma$, than the pure magnetic field.

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Figure captions

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Fig. 1 The Feynman diagram for the radiative neutrino transition in the presence of plasma and magnetic field.

Fig. 2 The dependence of the probability of the radiative neutrino transition $\nu_e \to \nu_e \gamma$ on energy in strongly magnetized cold plasma, $T \ll \mu$. The dotted lines correspond to $\nu_e \to \nu_e \gamma$ process in pure magnetic field. The lines 1, 2, 3 depict the probabilities for angles between the initial neutrino momentum and the magnetic field direction $\theta = \pi/4, \pi/2, 3\pi/4$ correspondingly. Here $W_0 = \alpha (G_F e B)^2 \mu / 8\pi^2$.

Fig. 3 The probability $\nu_e \to \nu_e \gamma$ process in strongly magnetized hot plasma, $T \gg \mu$. Here $W_0 = \alpha (G_F e B)^2 T / 8\pi^2$, other notations are the same as in the Fig. 2.

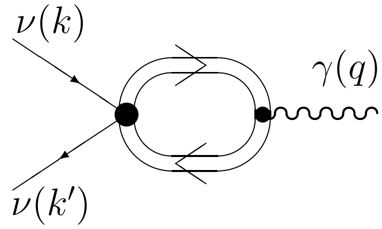

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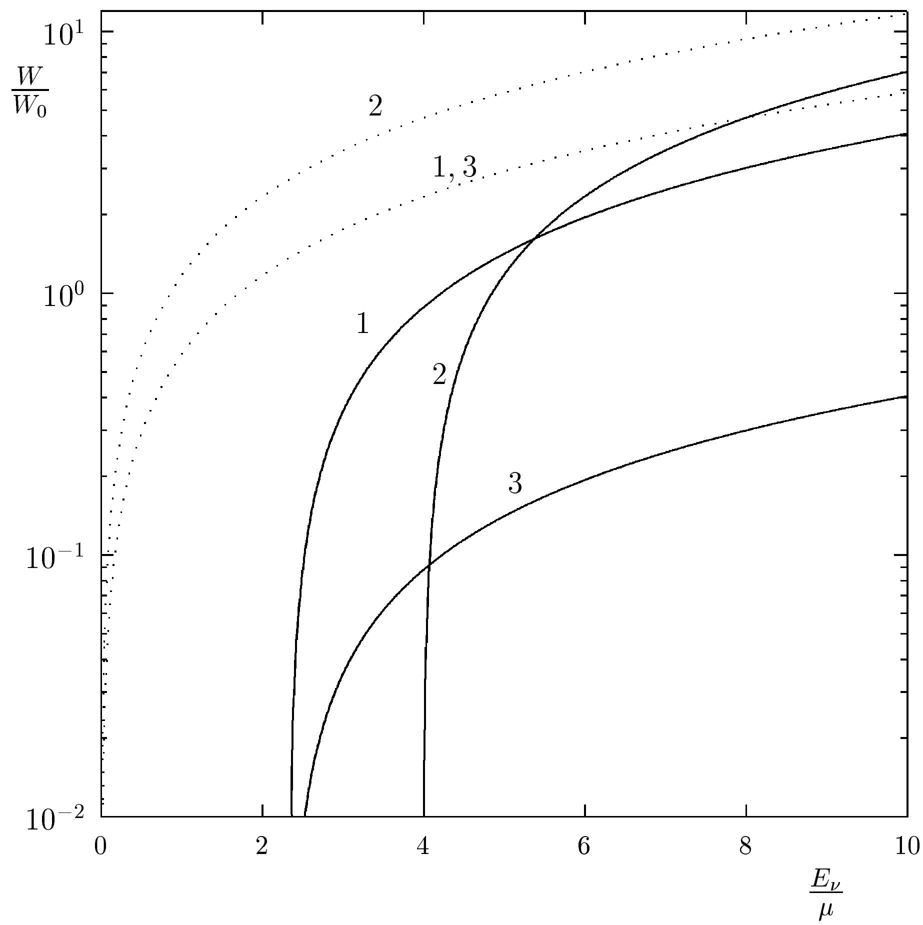
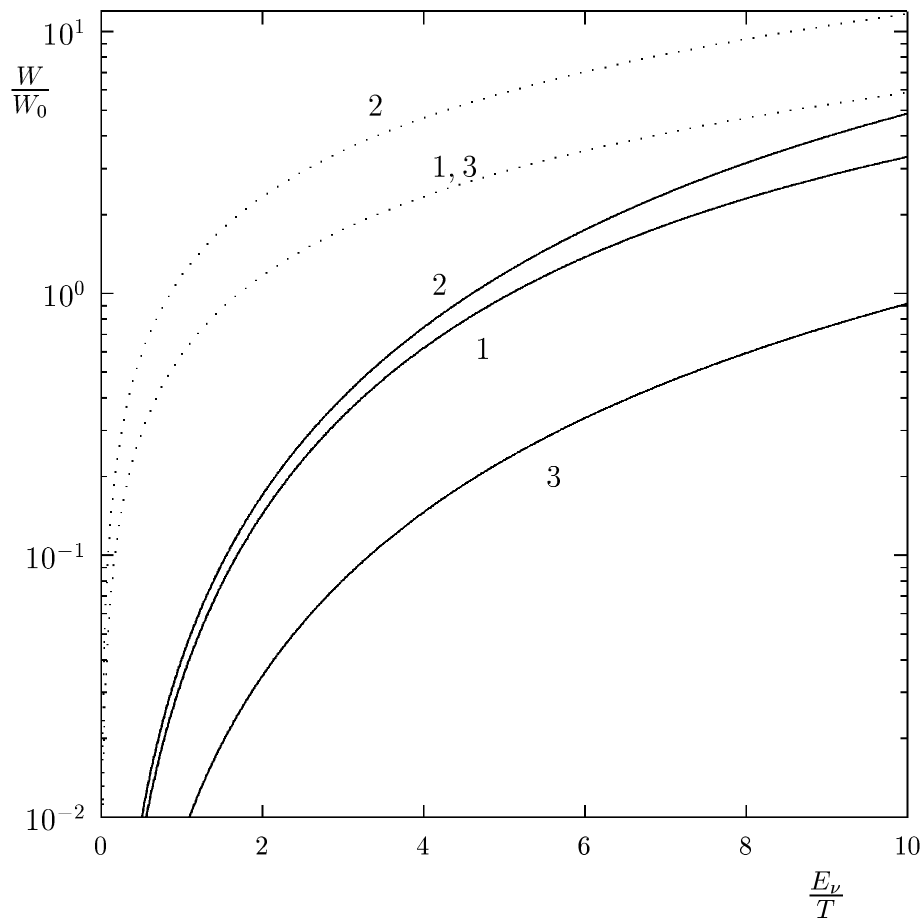
Figure 2: M.V. Chistyakov et al., “Radiative neutrino . . . ”

Figure 3: M.V. Chistyakov et al., “Radiative neutrino . . . ”



10^1
$\frac{W}{W_0}$
10^0
10^{-1}
10^{-2}
2
1, 3
1
2
3
0
2
4
6
8
10
$\frac{E_\nu}{\mu}$



$\frac{W}{W_0}$
10^1
10^0
10^{-1}
10^{-2}
0
2
4
6
8
10
$\frac{E_\nu}{T}$
2
1, 3
2
1
3